INTERACTION OF A SUBMERGED TURBULENT JET WITH A SOLID WALL

E. P. Volchkov, S. S. Kutateladze, and A. I. Leont'ev

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The turbulent layer of a wall jet has been analyzed in many theoretical and experimental studies [1-10]. Most theoretical investigations are based on the simultaneous solution of the equations of the turbulent jet and the boundary layer that forms at the wall. The differences lie in the methods used to correlate the velocity and temperature distributions, as well as in the friction and heat-transfer laws employed. In this article we present a method based on the further development of the idea of conservation of the laws governing wall turbulence with respect to change in boundary conditions.

1. Integral momentum relation. Consider a plane turbulent jet issuing from a slot and propagating along a smooth plane wall (Fig. 1) in a space filled with fluid of the same density. At the wall, beginning at the section x = 0, there forms a wall boundary layer of thickness δ_1 with a velocity maximum at its outer edge (turning point in the velocity profile $\partial w/\partial y = 0$).

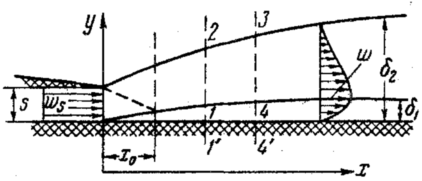

Fig. 1. Flow in a submerged wall jet.

The momentum equation for an element dx of thickness δ_2 is

$$\frac{d}{dx}\int_0^{\delta_2} \rho w^2\, dy = -\tau_w \tag{1.1}$$

or

$$\frac{d}{dx}\int_0^{\delta_1} \rho w^2 dy + \frac{d}{dx}\int_{\delta_1}^{\delta_2} \rho w^2 dy = -\tau_w. \tag{1.2}$$

Assuming that the shear stresses at the outer edge of the wall boundary layer are equal to zero (as $\partial w/\partial y = 0$ at $y = \delta_1$), we set up the momentum equation for the element 1-2-3-4:

$$\frac{d}{dx}\int_{\delta_1}^{\delta_2} \rho w^2 dy + w_0 \frac{d}{dx}\int_0^{\delta_1} \rho w\, dy = 0. \tag{1.3}$$

Hence it follows that

$$\frac{d}{dx}\int_{\delta_1}^{\delta_2} \rho w^2 dy = -w_0 \frac{d}{dx}\int_0^{\delta_1} \rho w\, dy. \tag{1.4}$$

With (1.4) taken into account, Eq. (1.2) becomes

$$w_0 \frac{d}{dx}\int_0^{\delta_1} \rho w\, dy - \frac{d}{dx}\int_0^{\delta_1} \rho w^2 dy = \tau_w. \tag{1.5}$$

We now introduce the characteristic parameters of the boundary layer – the momentum thickness δ^{**} and the displacement thickness δ^*, defined as

$$\delta^{**} = \int_0^{\delta_1} \frac{\rho w}{\rho_0 w_0}\left(1 - \frac{w}{w_0}\right) dy, \qquad \delta^* = \int_0^{\delta_1}\left(1 - \frac{\rho w}{\rho_0 w_0}\right) dy.$$

Equation (1.5) can be brought to the form

$$\tau_w = \frac{d}{dx}\rho_0 w_0^2 \delta^{**} + \rho_0 w_0 (\delta^* - \delta_1)\frac{dw_0}{dx} \tag{1.6}$$

or

$$\frac{dR^{**}}{dX} + \left[1 + \frac{\delta^*}{\delta^{**}} - \frac{\delta_1}{\delta^{**}}\right]\frac{R^{**}}{W_0}\frac{dW_0}{dX} = \frac{C_{f1}}{2} R_s W_0, \tag{1.7}$$

where

$$R^{**} = \frac{w_0 \delta^{**}}{\nu_0}, \quad X = \frac{x}{s}, \quad W_0 = \frac{w_0}{w_s}, \quad \frac{C_{f1}}{2} = \frac{\tau_w}{\rho_0 w_0^2}, \quad R_s = \frac{w_s s}{\nu_0}.$$

When the velocities in the wall boundary layer are distributed according to a 1/7 power law, we have

$$1 + \frac{\delta^*}{\delta^{**}} - \frac{\delta_1}{\delta^{**}} = C_1 = \text{const} = -8. \tag{1.8}$$

Equation (1.7) can also be obtained by integrating the boundary-layer equation of motion

$$w_x \frac{\partial w_x}{\partial x} + w_y \frac{\partial w_x}{\partial y} = \frac{\partial \tau}{\partial y} \tag{1.9}$$

over y from $y = 0$ to $y = \delta_1$, taking into account the continuity equation and the boundary conditions

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} = 0 \qquad \begin{array}{lll} w_x = w_y = 0, & \tau = \tau_w & \text{at } y = 0 \\ w_x = w_0 = f(x), & \tau = 0 & \text{at } y = \delta_1. \end{array} \tag{1.10}$$

2. Friction and heat transfer. We assume, as usual, that the friction in the wall boundary layer is described by the law

$$\frac{C_{f1}}{2} = \frac{\tau_w}{\rho_0 w_0^2} = \frac{A}{R^{**m}} \qquad (A = 0.0128,\ m = 0.25). \tag{2.1}$$

Here the values in parentheses are appropriate for a turbulent boundary layer [11] in the region of validity of the 1/7 power law. The momentum equation (1.7) becomes

$$\frac{dR^{**}}{dX} + C_1 \frac{R^{**}}{W_0}\frac{dW_0}{dX} = \frac{A}{R^{**m}} W_0 R_s. \tag{2.2}$$

It is known that $\delta_1 \ll \delta_2$ [1]. Therefore one can assume that the law of change of the maximum velocity in the wall jet remains practically the same as in the case of a free jet [1] with an initial cross-section 2S:

$$W_0 = C_2 X^a \approx 3.8 X^{-0.5}. \tag{2.3}$$

Now, integrating Eq. (2.2) from X_0 to X, we obtain

$$R^{**} = \left\{ R_0^{**(m+1)} \left(\frac{x_0}{x}\right)^{C_1 a(m+1)} + \frac{A(m+1) R_s C_2 X^{a+1}}{aC_1(m+1) + a + 1}\left[1 - \left(\frac{x_0}{x}\right)^{C_1 a(m+1)+a+1}\right]\right\}^{\frac{1}{m+1}}. \tag{2.4}$$

For $x \gg x_0$ we have

$$R^{**} \approx \left[\frac{A(m+1) R_s C_2 X^{a+1}}{aC_1(m+1) + a + 1}\right]^{\frac{1}{m+1}}. \tag{2.5}$$

Substituting R^{**} into (2.1), we obtain the friction coefficients

$$\frac{\tau_w}{\rho_0 w_0^2} = \frac{C_{f1}}{2} = \frac{0.0315}{R_s^{0.2} X^{0.1}} \qquad \left(C_{f1} = 0.0825\left[\frac{w_0 x}{\nu_0}\right]^{-0.2}\right) \tag{2.6}$$

$$\frac{\tau_w}{\rho_0 w_s^2} = \frac{C_{f2}}{2} = \frac{0.457}{R_s^{0.2} X^{1.1}}. \tag{2.7}$$

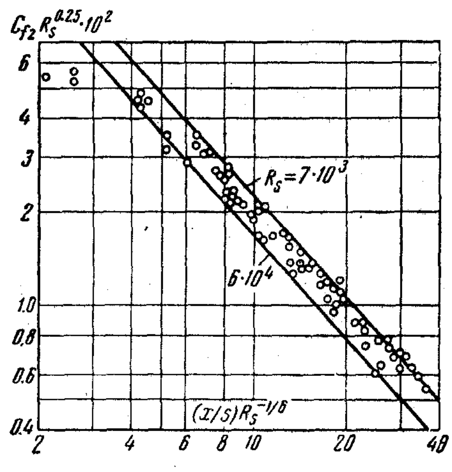

Fig. 2. Continuous lines: wall friction coefficient in a submerged wall jet according to Eq. (2.7). Experimental points: from Myers et al. [4].

Figure 2 compares the results of Eqs. (2.7) (continuous lines) with the experimental data of Myers et al. [4]. One can see that the theory is in satisfactory agreement with the experimental data. Sigalla [9] has found an empirical correlation for the wall friction coefficient for $X > 30$,

$$C_{f1} = 0.0865 \left(\frac{w_0 x}{\nu}\right)^{-0.2}, \tag{2.8}$$

which differs from Eq. (2.6) by only 5%. Seban and Back [7] have found that in the case of a jet injected into a uniform flow, with jet to free stream mass velocity ratio

$$3 < (\rho w)_s / (\rho w)_\infty < 9,$$

the velocity w_0 changes according to the law

$$w_0 / w_s = 3.6 X^{-0.45} . \qquad (2.9)$$

Using the law of variation of velocity and Eqs. (2.1) and (2.5), we obtain the friction coefficient for a wall jet injected into a weak free stream

$$\frac{C_{f1}}{2} = \frac{0.0314}{R_s^{0.2} X^{0.11}} . \qquad (2.10)$$

As can be seen in Fig. 3, Eq. (2.10) is in satisfactory agreement with the experimental data [7].

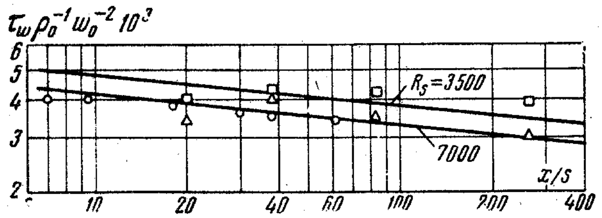

Fig. 3. Continuous lines: wall friction in the presence of a weak free stream according to (2.10). Experimental points: from Seban and Back [7].

With the functional dependence of the Stanton number

$$S = {}^1/_2 C_f P^{-0.6}$$

(where P is the Prandtl number) taken into account, Eqs. (2.6) and (2.10) yield:

a) Stanton and Nusselt numbers for a submerged wall jet

$$S_2 = \frac{\alpha}{\rho_0 w_s C_p} = \frac{0.12}{R_s^{0.2} X^{0.6} P^{0.6}} , \qquad (2.11)$$

$$N_x = \frac{\alpha x}{\lambda} = 0.1197 \left(\frac{w_s x}{\nu_0}\right)^{0.8} X^{-0.4} P^{0.4} , \qquad (2.12)$$

b) Stanton number for a wall jet injected into a weak stream

$$\frac{(\rho w)_s}{(\rho w)_\infty} > 3, \qquad S_2 = \frac{0.113}{R_s^{0.2} X^{0.56} P^{0.6}} . \qquad (2.13)$$

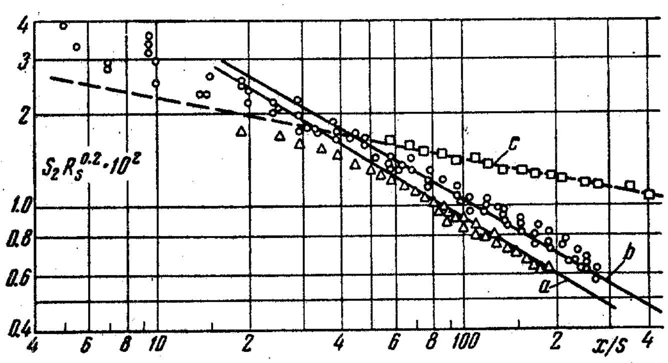

Fig. 4. Heat-transfer coefficient in a wall jet. a, b, c) Computed from Eqs. (2.11), (2.13), (2.14). Triangles: experiments [5] at t_w = const; circles: experiments [6, 7] at q_w = const, $3 < (\rho w)_s / (\rho w)_\infty < 9$; squares: experiments [6] at q_w = const, $1.05 < (\rho w)_s / (\rho w)_\infty < 1.1$.

Figure 4 compares the results obtained from (2. 11) and (2. 13) with the experimental data of Myers et al. [5] and Seban and Back [6, 7]. The same figure also shows Seban's [6] experimental data for the case $(\rho w)_s/(\rho w)_\infty \approx 1$, which are well correlated by the usual equation

$$S_2 = 0.0288 R_x^{-0.2} P^{-0.6}. \qquad (2.14)$$

Jakob et al. [10] found the experimental correlation

$$N_x = \frac{\alpha x}{\lambda} = 0.105 \left(\frac{w_s x}{\nu}\right)^{0.8} X^{-0.4}, \qquad (2.15)$$

which is identical with (2. 13) when P = 0. 71.

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